THE METHOD OF COMPUTATION in MDPREF[1]

- **Given N subjects and p stimuli, we define a first-score matrix S (N x p)**

  $$S = \{ s(i,j) \}$$

  of N subjects' ratings or rankings of the p stimuli (<u>the data</u>).

- **The <u>solution</u>, consists of the two solution matrices**

$$X = \{x(i,a)\} \quad i = 1,2,\ldots N; \quad a = 1,2 \ldots. r)$$

(a configuration of N subject <u>vectors</u> in an r- dimensional space), and

$$Y = \{y(j,a\}\} \quad J = 1,2,\ldots p; \quad a = 1,2,\ldots r$$

(a configuration of p stimulus <u>points</u> an r-dimensional space).

- **The data are related to the solution by means of a (fitted) <u>second-score</u> score matrix, S* (N x p):**

  $$S^* = \{ s^*(i,j) \} = X.Y, \text{ and}$$

  $$S^* \approx S \quad \text{(i.e. } S^* \text{ is a LS fit to S)}$$

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[1]This is based on Carroll (1964); see also MDS(X) Users' Guide and User Manual

- **The <u>solution</u> is obtained by factoring (singular-value decomposition) s.t.**

$$S^* = U \beta V'$$

$$= U_r \beta_r V'_r$$

consisting of the first r columns of U and of V' respectively.

The solution-matrices are then given as :

$$X = U_r \beta_r \text{; and } Y = \beta_r V_r \text{ resp.}$$

- **This is an <u>Eckart-Young factorization</u>:**

  U is a matrix with eigenvectors of SS' as its cols.

  V has as its columns the eigenvectors of S'S

  β is a diagonal matrix of the corresponding eigenvalues, λ (j).

- **If the eigenvalues are ordered acording to decreasing size, then X and Y (of rank *r*) give the best LS approximation to S*.**